

The article presents the results of the numerical solution of the nonlinear problem of heating a medium by optical radiation.

The question of the destruction threshold of transparent solid materials by optical radiation has been studied in great detail. It is generally agreed that in most cases of practical interest, damage is caused by heating of the medium near absorbing microinclusions (metal particles) or other inhomogeneities of the material [1-5]. Depending on the pulse duration of the radiation and its intensity, a certain type of inhomogeneity proves to be decisive in the process of destruction. In the case of absorption of light near metal inclusions of 1  $\mu\text{m}$  in size to sizes visible to the naked eye, a modification of the dimensional effect (which is also meant everywhere in the present article) takes place which manifests itself in the fact that for each intensity and duration of the radiation pulse there is a critical size contributing decisively to the process of destruction of the material.

Microinclusions and inhomogeneities of the medium smaller than tenths of a micrometer are difficult to check, and their properties in actual systems have been insufficiently studied. It is assumed that when the number of such inhomogeneities is large, their contribution may lead to uniform heating of the medium as a whole; i.e., a certain change of the mean absorption coefficient of the material, caused by the inhomogeneities, occurs. This type of inhomogeneity is determined by the production technology of the material, and in actual media there are always such inhomogeneities. They may be admixtures of a heterogeneous substance with molecular dimensions. It may be assumed that there is a certain mean concentration of such admixture particles, and that the concentration fluctuates relative to a mean value, this fluctuation leading to fluctuations of the mean absorption coefficient within the material.

Assuming that the inhomogeneities are an admixture of molecular dimensions of fairly high concentration, we may take it that the mean distance between particles of the admixture is much smaller than the characteristic dimensions of the irradiated zone, and that the concentration of admixture is a practically continuous function of the coordinates. Under these conditions, fluctuations of concentration manifest themselves as homogeneous formations whose dimensions are much larger than molecular. Calculation of the heating of the medium in the range of this type of fluctuation of the absorption coefficient, taking into account the nonlinear dependence of the absorption coefficient on the temperature (pressure), shows that it is possible that the temperature increases intensely (exponentially) [6]. How do the dimensions of the fluctuation manifest themselves under these conditions? To answer this question, we must proceed from a certain model of the change of the properties of the medium when the corresponding dimensions change.

The existence of a mean concentration of admixture particles that is relatively constant in respect to volume leads to the requirement that the full number of admixture particles in the characteristic volume be constant. If we examine a constant volume  $\alpha_0^3$ , inside which there arises fluctuation of the absorption coefficient, and if we assume that there is an exponential dependence of the concentration of the admixture particles in transition from one level to another  $n(r) = n_1 \theta(\alpha - r)$ , we arrive at the requirement of a correlation between the dimensions of the fluctuation  $\alpha$  and the concentration of the admixture  $n_1$  inside the zone with increased absorption coefficient. The total number  $N$  of admixtures within the volume  $\alpha_0^3$  is equal to  $4\pi\alpha^3 n_1 / 3$ , therefore  $n_1 = 3N / (4\pi\alpha^3)$ . Taking into account that the absorption coefficient contains a component that is proportional to the concentration of the admixtures, we

arrive at a correlation between the absorption coefficient  $\chi_1$  in the volume of the fluctuation and the dimensions of the volume. In reality the corresponding dependence is not strictly inversely proportional to the third power of the dimension, because it is possible that an admixture is present in the surrounding depleted layer. Transition in space from the periphery of fluctuation to the center need not lead to a jumplike change in concentration. However, taking into account the complexity of the structure of materials such as glass, the difference in solubility of the admixture in different phases, etc., we may assume that the examined model corresponds qualitatively to reality. Other cases are possible, too, and they will also be examined. In particular, a calculation was carried out for changing dimension but with constant concentration of the admixtures, and also for constant dimension but changing concentration.

In order to shed light on the principal features of the processes on material being heated by optical radiation, calculations analogous to [6] were carried out. We study the case of spherically symmetrical fluctuation of the absorption coefficient engendered by non-uniform distribution of the admixtures. We take into account the temperature redistribution as a result of the relatively slow-acting mechanism of heat conduction and the development of considerable thermoelastic stresses accompanying the change in the temperature field; here the lag of the change of the stress field upon change of temperature is assumed to be negligibly small. This enables us to consider the problem of calculating thermoelastic stresses to be quasi-steady-state, and to write and solve the equation of heat conduction for the temperature, taking into account the heating by optical radiation and the dependence of the parameters of the medium on the developing rather large thermoelastic stresses. We assume that the medium is a dielectric for which the absorption coefficient depends exponentially on the ratio of the characteristic energy to the temperature [4]. The characteristic energy may be taken to be equal to the width of the forbidden band, but then some correction must be envisaged. According to experimental data, the width of the forbidden band depends on the pressure, and within a certain pressure range this dependence may be approximated by a linear one. Several authors [7, 8] showed that if a dependence of the kind of [4] is to be used for the absorption coefficient of the medium, the nonlinear dependence of E on the temperature has to be taken into account. In particular, Morichev et al. [7, 8] showed that for different ranges of T, E has to be different, and the deviation of E from the width of the forbidden band toward lower values has to be taken into account. In the present work we assume that two sections of the T scale ( $T < 6T_0$ ,  $T > 6T_0$ ) exist for which  $E_0/(2kT_0) = 5$ ,  $E_0/(2kT_0) = 60$ . When  $T = 6T_0$ , both dependences are joined; when  $T = T_0$ , the value of  $\chi$  corresponds to typical media at normal conditions.

Calculation of the thermoelastic stresses for the case of spherical symmetry can be carried out by known methods developed for the steady-state temperature field [9]. In the equation of heat conduction, the absorption coefficient is assumed to consist of two components, one of which relates to the medium, and the other to the admixtures. In view of this, the latter is proportional to the concentration of the admixture. We write the equation of heat conduction

$$\rho c \frac{\partial T}{\partial t} = \frac{\chi}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + I\chi(r, T). \quad (1)$$

The absorption coefficient in (1) is adopted in the form

$$\chi(r, t) = \chi_1(r) + \chi_0 \exp \left\{ -\frac{1}{2kT} \left( E_0 - \frac{2\beta E_1 \alpha}{3(1-\sigma)} \frac{1}{r^3} \int_0^r r_1^2 (T(r_1) - T_0) dr_1 \right) \right\}. \quad (2)$$

The first component in  $\chi(r, T)$  is determined by absorption by the admixtures, the second one by absorption by the medium; moreover, as was noted before, the second component depends additionally on the temperature.

The function  $\chi_1(r)$  has the form  $\chi_1(r) = \chi_1 \Theta(a-r)$ , where  $\Theta(a-r)$  is a step function. The last term in the exponent takes into account the narrowing of the forbidden band because of thermoelastic stresses, and  $\beta$  is the corresponding proportionality factor.

Equation (1) does not take into account the dependence of  $\chi$  on the temperature, since it is of the same origin as the dependence  $\chi(T)$ . In calculations the zones in which the temperature is sufficiently high for the dependence of  $\chi$  on T to manifest itself substantially have small dimensions, and the result is that the dependence  $\chi(T)$  has little effect on the

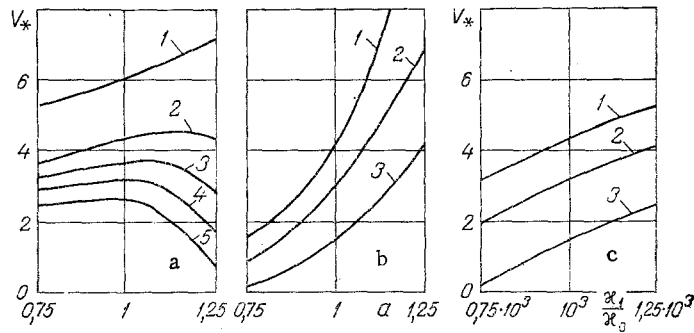


Fig. 1. Dependence of  $V_*$  on  $\alpha$  (a), (b) and on  $\kappa_1/\kappa_0$  (c): a)  $\kappa_1 \sim \alpha^{-3}$ : 1)  $T_* = 2T_0$ ; 2) 2.5; 3) 2.75; 4) 3; 5)  $T_* = 3.25 T_0$  ( $\alpha, 10^{-6}$  m); b)  $\kappa_1 = \text{const}$ : 1)  $T_* = 2.5 T_0$ ; 2) 3; 3) 4; c)  $\alpha = 10^{-6}$  m: 1)  $2.5 T_0$ ; 2) 3; 3)  $4 T_0$ .

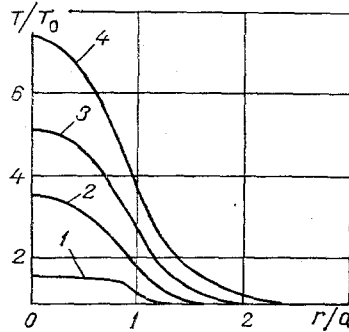


Fig. 2. Distribution of  $T/T_0$  in the medium for the case  $\alpha = 10^{-6}$  m,  $\kappa_1 = 10^3 \kappa_0$ : 1)  $t = 0.06 \cdot 10^{-6}$  sec; 2)  $t = 0.3 \cdot 10^{-6}$  sec; 3)  $t = 0.6 \cdot 10^{-6}$  sec; 4)  $t = 1.2 \cdot 10^{-6}$  sec.

general behavior of the solution of Eq. (1). Therefore, to avoid complications of the calculations,  $\chi$  was assumed to be constant.

The thermoelastic part of the problem also yields values of displacement of the points of the medium upon development of stresses in the form

$$u(r) = \alpha \frac{1 + \sigma}{3(1 - \sigma)} \frac{1}{r^2} \int_0^r r_1^2 (T(r_1) - T_0) dr_1, \quad (3)$$

where  $T_0$  is the temperature of the medium at infinity. We carried out the numerical solution of Eqs. (1), (2) for a number of values of the dimension of fluctuation and concentration of admixture particles. The graphs of three of the above-noted cases are shown in the figures.

In the calculations we assumed the following values of the parameters of the system:  $\rho = 3 \cdot 10^3$  kg/m<sup>3</sup>,  $c = 1.3 \cdot 10^3$  J/kg  $\cdot$   $^\circ$ K,  $\kappa_0 = 0.1$  m<sup>-1</sup>,  $\chi = 1.3$  W/m  $\cdot$   $^\circ$ K,  $\alpha = 0.008/T_0$ ,  $\sigma = 0.22$ ,  $\beta = 0.0001 \cdot 2kT_0/p_0$ ,  $E_1 = 7 \cdot 10^5 p_0$ ,  $p_0 = 10^5$  Pa,  $T_0 = 300^\circ$ K. The intensity was taken equal to  $10^{14}$  W/m<sup>2</sup>. The dimensions of inhomogeneity lie within the interval of values  $0.75 \cdot 10^{-6}$  to  $1.25 \cdot 10^{-6}$  m. The concentration of the admixture with constant  $\alpha$  then changes in the interval of values that corresponds to the change of  $\kappa_1$  within the limits (750-1370)  $\kappa_0$ . Figure 1 shows the behavior of the volume  $V_*$  within which the temperature in the medium exceeds some value  $T_*$ . Different values of  $T_*$  may correspond to the thresholds of different physical changes in the medium. The duration of the examined pulse  $\tau$  was taken equal to  $0.6 \cdot 10^{-6}$  sec.

In the case of Fig. 1a, the concentration  $n_1$  is such that with  $\alpha = 10^{-6}$  m,  $\kappa_1 = 10^3 \kappa_0$ . The value of  $\alpha$  changes within the interval  $0.75 \cdot 10^{-6}$ - $1.25 \cdot 10^{-6}$  m; an increase of  $V_*$  is observed. For a lower value of  $T_*$  the curve  $V_*$  in the given interval of  $\alpha$  increases monotonically.

cally. For higher temperatures  $T_*$  we find that the curve passes through its maximum, and then  $V_*$  drops. For  $T_* = 3.25 T_0$  the curve at the boundary of the interval drops almost to zero. Such a course of the curves indicates that there possibly exists a critical dimension of the fluctuations in relation to the threshold of destruction of the material.

In the case of Fig. 1b we find monotonic increase of the curves with increasing  $\alpha$ . The dependence is nonlinear, and for the temperature  $T_* = 4T_0$  the initial value of  $V_*$  is practically equal to zero. This indicates that there exists a minimum dimension of the fluctuation leading to the destruction of the material at the given intensity and duration of the pulse.

In the case of Fig. 1c, the curves increase monotonically but the rate of increase becomes smaller with increasing concentration; this indicates the possibility of saturation. For  $T_* = 4T_0$  the initial value of the curve is practically equal to zero; this indicates the existence of a minimum concentration at which destruction of the material occurs.

Figure 2 illustrates the dynamics of temperature increase in the medium. At the initial instants, the temperature distribution copies fairly accurately the distribution of admixture concentration in the medium. This circumstance may be made use of for determining experimentally the distribution of the admixture concentration. With sufficiently sensitive methods of finding the temperature distribution in transparent material, heating with fairly weak, short pulses makes the distribution of the admixture visible. The quantitative comparison of concentration and temperature may be assisted by calculations of the type carried out above. At the later instants (Fig. 2) the temperature distribution differs considerably from the distribution of the concentration.

The calculations that were carried out indicate that it is possible to compare the temperature distribution in a medium upon irradiation by fluxes whose intensity lies substantially below the threshold value, with the distribution of the concentration. The data presented in Figs. 1, 2 characterize the possible dependences of the volume of the zone in which the parameters attain critical values on the dimensions and amplitude of the fluctuations of the admixture concentration. The corresponding regularities are important in determining the threshold of destruction of material by optical radiation from the known initial distribution of the concentration of absorbing admixtures. The values of the parameters used in the calculations were close to the experimental ones.

#### NOTATION

$a_0, a$ , characteristic dimensions;  $n(r), n_1$ , concentration of admixture;  $\theta$ , step function;  $r, r_1$ , radial coordinate;  $N$ , number of admixtures;  $T, T_*, T_0$ , temperature;  $E, E_0$ , characteristic energy;  $\kappa, \kappa_0, \kappa_1$ , absorption coefficient;  $k$ , Boltzmann constant;  $\rho$ , density of the medium;  $c$ , specific heat capacity of the medium;  $t$ , time;  $\chi$ , thermal conductivity of the medium;  $I$ , intensity of the luminous flux;  $E_1$ , Young's modulus;  $\alpha$ , coefficient of volume expansion;  $\sigma$ , Poisson ratio;  $u(r)$ , displacement of the points of the medium;  $V_*$ , characteristic volume;  $\tau$ , pulse duration.

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TEMPERATURE DEPENDENCE OF THE ELECTRICAL RESISTIVITY AND NORMAL  
INTEGRAL COEFFICIENT OF RADIATION OF METALS

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On the basis of an analysis of the values of the normal integral coefficient of radiation (radiative capacity) and dc resistivity of metals, we obtain formulas describing the temperature dependence of these quantities and their interrelationship over a wide range of temperatures.

A great deal of information has already been accumulated concerning the temperature dependence of the electrical and radiative properties of metals. However, in a number of cases the experimental data found in different literature sources are not in good agreement. Consequently, the available information must be generalized in a way that will make it possible to establish the degree of reliability required for the correct solution of practical problems and will provide a method for its adequate theoretical interpretation.

The purpose of the present article is to determine — on the basis of a generalization of the results of an experimental investigation of the electrical resistivity ( $\rho_{\mu}$ ) and the integral normal coefficient of radiation ( $\epsilon_{tn}$ ) of metals — the characteristics of the temperature-dependent variation of these quantities at temperatures below the melting point.

We carried out an analysis of the experimental data for  $\epsilon_{tn}$  and  $\rho_{\mu}$  taken from various literature sources [1-5, 12-14], as well as the values of  $\epsilon_{tn}$  calculated from the results of an investigation of the optical constants of metals [6] and from the integral hemispherical radiative capacity ( $\epsilon_{th}$ ) by means of the formulas [7]:

$$\frac{\epsilon_{tn}}{\epsilon_{th}} = 1 + \exp[-(5.2\epsilon_{tn} + 1.16)], \quad \epsilon_{tn} < 0.2; \quad (1)$$

$$\frac{\epsilon_{tn}}{\epsilon_{th}} = 7.37 \cdot 10^{-2} + \exp[-(0.11 \ln \epsilon_{tn} + 0.14)], \quad \epsilon_{tn} \geq 0.2. \quad (2)$$

In Fig. 1, where the values of  $\rho_{\mu}$  and  $\epsilon_{tn}$  for a number of the metals we considered are plotted against temperature, it can be seen that a change in the temperature dependence of these quantities in some metals, e.g., in nickel (see Fig. 1c), takes place at the same temperature. In other metals, in particular in tungsten (Fig. 1a), the relation  $\epsilon_{tn} = f(T)$  changes at a temperature that lies in the interval 1800-1900°K, while the function  $\rho_{\mu} = \varphi(T)$  remains unchanged over the entire temperature range represented in the figure. This fact contradicts the conclusion drawn in [8] to the effect that the variation of  $\rho_{\mu}(T)$  for metals as the temperature varies is similar to that of  $\epsilon_{tn}(T)$ .

For most metals the experimental values of  $\rho_{\mu}$  and  $\epsilon_{tn}$  were obtained at temperatures lower than the melting point  $T^*$ , which, as is known, is a characteristic quantity for each specific metal. Obviously,  $\rho_{\mu}^*$  and  $\epsilon_{tn}^*$ , corresponding to the solid phase of the metal at the melting point, can be characteristic quantities of the same kind.

Since the temperature dependence of the electrical resistivity of metals, as of all crystalline solids, remains unchanged in the range of high temperatures up to the melting point [9], the numerical value of  $\rho_{\mu}^*$  for each metal can be determined from the high-temperature segment of curves of the type represented in Fig. 1 by extrapolating them to  $T = T^*$ . The numerical value of  $\epsilon_{tn}^*$  is determined in the same way.

We found that the electrical resistivity, referred to the appropriate absolute temperature and expressed in terms of its characteristic quantity and the melting point, is a power function of the relative temperature:

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